4. J. N. Johnson, "Dynamic fracture and spallation in ductile solids," J. Appl. Phys., 52, No. 4 (1981).
5. V. K. Golubev, S. A. Novikov, et al., "Influence of temperature on the critical conditions for spall rupture of metals," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1980).
6. V. K. Golubev, S. A. Novikov, et al., "On the mechanisms of spall rupture of steel St. 3 and 12Kh18N10T in the −196...800°C temperature range," Probl. Prochn., No. 5 (1981).
7. V. G. Grigor'ev, S. Z. Dunin, and V. V. Surkov, "Collapse of a spherical pore in a viscoplastic material," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 1 (1981).
8. L. Seaman, D. S. Curran, and D. A. Shockey, "Computational models for ductile and brittle fracture," J. Appl. Phys., 47, No. 11 (1976).
9. V. S. Ivanova, Metal Rupture [in Russian], Metallurgiya, Moscow (1979).
10. L. E. Murr, "Work hardening and the pressure dependence of dislocation density arrangements in shock-loaded nickel and copper," Scripta Metall., 12, No. 2 (1978).
11. L. Davison and R. A. Graham, "Shock compression of solids, Phys. Reports, 55, No. 4 (1979).
12. B. M. Butcher, M. M. Carroll, and A. C. Holt, "Shock-wave compaction of porous aluminum," J. Appl. Phys., 45, No. 9 (1974).
13. L. C. Chhabildas and J. R. Asay, "Rise-time measurements of shock transitions in aluminum, copper, and steel," J. Appl. Phys., 50, No. 4 (1979).
14. V. I. Al'shits and V. L. Indenbom, "Dynamic retardation of dislocations," Usp. Fiz. Nauk, 115, No. 1 (1975).

BENDING OF AN ANISOTROPIC PLATE CONTAINING AN ANISOTROPIC ELASTIC INCLUSION

M. I. Zadvornyak and T. L. Martynovich

UDC 539.3

A thin plate of thickness h is considered that has a curvilinear hole into which is soldered an elastic body made of another material. The plate and the inclusion have rectilinear anisotropy with respect to the elastic properties of the material and at each point have a plane of elastic symmetry parallel to the median plane xOy. The principal elasticity directions for the plate and inclusion are at an angle φ (Fig. 1). The line L dividing the regions $S^{(1)}$ and $S^{(2)}$ corresponding to the different anisotropic materials is described by an equation of the form

$$t = x + iy = R\left(e^{i\theta} + \sum_{k=1}^{N} C_k e^{-ik\theta}\right), \quad \sum_{k=1}^{N} k\,|C_k|^2 < 1. \tag{1}$$

Along line L between regions $S^{(\alpha)}$ (α = 1, 2), the conjugation conditions should apply:

$$M_n^{(1)} = M_n^{(2)},\ N_n^{(1)} + \frac{\partial H_{n\tau}^{(1)}}{\partial s} = N_n^{(2)} + \frac{\partial H_{n\tau}^{(2)}}{\partial s},$$
$$W^{(1)} = W^{(2)},\ \frac{\partial W^{(1)}}{\partial n} = \frac{\partial W^{(2)}}{\partial n}, \tag{2}$$

while in parts of the plate remote from the inclusion the bending and torsional moments are bounded: $M^\infty_x = M_1$, $M^\infty_y = M_2$, $M^\infty_{xy} = H_{12}$. There are no external localized forces and distributed loads normal to the median plane in the regions $S^{(\alpha)}$ (α = 1, 2). Here n and τ are the normal and tangent to line L.

In the analytic solution, the region $S^{(1)}$ will be considered as infinite (the perturbation in the elastic state of the plate due to the inclusion does not attain the outer boundary of the plate).

Lvov. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 165-171, November-December, 1983. Original article submitted August 23, 1982.

 0021-8944/83/2406-0920$07.50

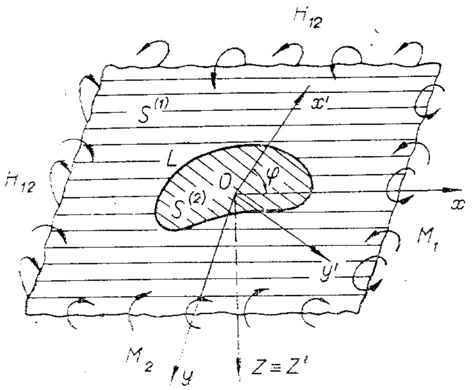

Fig. 1

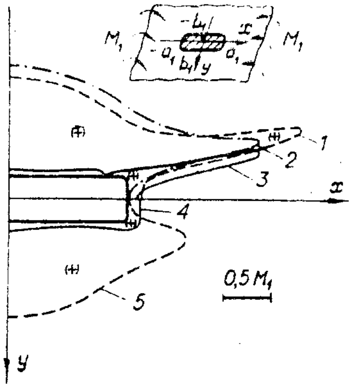

Fig. 2

The formulas from the theory of bending for anisotropic plates [1, 2] enable us to represent (2) as integral relations containing an arbitrary function F(z) holomorphic in $S^{(1)}$ (or $S^{(2)}$) [3]:

$$\int_L F(t)\,dV^{(1)} = \int_L F(t)\,dV^{(2)} + iC\int_L F(t)\,dt,$$
$$\int_L \overline{F(t)}\,dV^{(1)} = \int_L \overline{F(t)}\,dV^{(2)} + iC\int_L \overline{F(t)}\,dt,$$
$$\int_L F(t)\,dU^{(1)} = \int_L F(t)\,dU^{(2)},\quad \int_L \overline{F(t)}\,dU^{(1)} = \int_L \overline{F(t)}\,dU^{(2)}, \tag{3}$$

where

$$dV^{(\alpha)} = \sum_{j=1}^{2}\left[\left(q_j^{(\alpha)} + i\frac{p_j^{(\alpha)}}{\mu_j^{(\alpha)}}\right)\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right)dz_j^{(\alpha)} + \left(\bar{q}_j^{(\alpha)} + i\frac{\bar{p}_j^{(\alpha)}}{\bar{\mu}_j^{(\alpha)}}\right)\overline{\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right)}\,d\bar{z}_j^{(\alpha)}\right], \tag{4}$$
$$dU^{(\alpha)} = \sum_{j=1}^{2}\left[\left(1 + i\mu_j^{(\alpha)}\right)\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right)dz_j^{(\alpha)} + \left(1 + i\bar{\mu}_j^{(\alpha)}\right)\overline{\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right)}\,d\bar{z}_j^{(\alpha)}\right].$$

Here t is the affix for a point on contour L, while $z^{(\alpha)}_j = x + \mu^{(\alpha)}_j y$ (j, α = 1, 2) are generalized complex variables, which vary in the regions $S^{(\alpha)}_j$, obtained from the regions $S^{(\alpha)}$ by appropriate affine transformations, while $\mu^{(\alpha)}_j = \alpha^{(\alpha)}_j + i\beta^{(\alpha)}_j$ are the roots of the corresponding characteristic equations, $p^{(\alpha)}_j$, $q^{(\alpha)}_j$ are certain constants [1, 2], $\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right) = \varphi_j^{(\alpha)\prime}\left(z_j^{(\alpha)}\right)$ are analytic functions describing the state of stress in the plate, and C is a real constant.

The contours for the regions $S^{(\alpha)}_j$ of the variables $z^{(\alpha)}_j$ are denoted by $L^{(\alpha)}_j$, while the affixes for the points are denoted by $t^{(\alpha)}_j$. Between the affixes for the points on $L^{(\alpha)}_j$ and L there exists the affine correspondence

$$t_j^{(\alpha)} = \frac{1}{2}\left(1 - i\mu_j^{(\alpha)}\right)t + \frac{1}{2}\left(1 + i\mu_j^{(\alpha)}\right)t \quad (j,\ \alpha = 1,\ 2). \tag{5}$$

The real constant C appearing in (3) is determined from the condition for uniqueness in the plate deflection

$$2\,\mathrm{Re}\left[\sum_{j=1}^{2}\int_{L_j^{(\alpha)}} \varphi_j^{(\alpha)}\left(t_j^{(\alpha)}\right)dt_j^{(\alpha)}\right] = 0.$$

The analytic functions $\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right) = \varphi_j^{(\alpha)\prime}\left(z_j^{(\alpha)}\right)(j, \alpha = 1,\ 2)$ should satisfy the conditions for uniqueness of the displacements

$$\int_{L_j^{(\alpha)}} \Phi_j^{(\alpha)}\left(t_j^{(\alpha)}\right)dt_j^{(\alpha)} = D_1^{(\alpha j)}\left(M_x^{(\alpha)} - yP_z^{(\alpha)}\right) + D_2^{(\alpha j)}\left(M_y^{(\alpha)} + xP_z^{(\alpha)}\right) \quad (j = 1,\ 2), \tag{6}$$

where $M^{(\alpha)}_x$, $M^{(\alpha)}_y$ are the components of the principal moments and $P^{(\alpha)}_z$ is the principal vector for the forces applied to the contour L of the regions $S^{(\alpha)}$ (α = 1, 2). The quantities $D^{(\alpha j)}_k$ (k, α, j = 1, 2) are calculated from the following formulas:

$$D_1^{(\alpha 1)} = D_1^{(\alpha 1)}\left(\mu_1^{(\alpha)}, \bar{\mu}_1^{(\alpha)}, \mu_2^{(\alpha)}, \bar{\mu}_2^{(\alpha)}\right) = \frac{\mu_1^{(\alpha)}\left(\mu_2^{(\alpha)} - \bar{\mu}_2^{(\alpha)}\right)}{4D_{22}^{(\alpha)}\beta_1^{(\alpha)}\beta_2^{(\alpha)}\left(\mu_2^{(\alpha)} - \mu_1^{(\alpha)}\right)\left(\bar{\mu}_2^{(\alpha)} - \mu_1^{(\alpha)}\right)},$$

$$D_2^{(\alpha 1)} = D_2^{(\alpha 1)}\left(\mu_1^{(\alpha)}, \bar{\mu}_1^{(\alpha)}, \mu_2^{(\alpha)}, \bar{\mu}_2^{(\alpha)}\right) = -\frac{|\mu_1^{(\alpha)}|^2 |\mu_2^{(\alpha)}|^2\left(\mu_2^{(\alpha)} - \bar{\mu}_2^{(\alpha)}\right)}{4D_{11}^{(\alpha)}\beta_1^{(\alpha)}\beta_2^{(\alpha)}\left(\mu_2^{(\alpha)} - \mu_1^{(\alpha)}\right)\left(\bar{\mu}_2^{(\alpha)} - \mu_1^{(\alpha)}\right)},$$

$$D_1^{(\alpha 2)} = D_1^{(\alpha 1)}\left(\mu_2^{(\alpha)}, \bar{\mu}_2^{(\alpha)}, \mu_1^{(\alpha)}, \bar{\mu}_1^{(\alpha)}\right), \quad D_2^{(\alpha 2)} = D_2^{(\alpha 1)}\left(\mu_2^{(\alpha)}, \bar{\mu}_2^{(\alpha)}, \mu_1^{(\alpha)}, \bar{\mu}_1^{(\alpha)}\right).$$

For large $|z^{(1)}_j|$, the functions $\Phi^{(1)}_j$ $(z^{(1)}_j)$ take the following form (principal vector $P^{(\alpha)}_z = 0$):

$$\Phi_j^{(1)}\left(z_j^{(1)}\right) = A_j^{(1)} + D^{(1j)} z_j^{(1)-1} + o\left(z_j^{(1)-2}\right) \quad \left(z_j^{(1)} \in S_j^{(1)}, \quad j = 1, 2\right).$$

The constants $D^{(1j)}$ are found from the conditions for uniqueness in the displacements of (6), while the coefficients $A^{(1)}_j$ are related to the bending and torsional moments in the infinitely remote parts of the plate by [2]

$$M_x^{\infty} = -2\,\mathrm{Re}\sum_{j=1}^{2} p_j^{(1)} A_j^{(1)}, \quad M_y^{\infty} = -2\mathrm{Re}\sum_{j=1}^{2} q_j^{(1)} A_j^{(1)},$$

$$H_{xy}^{\infty} = -2\mathrm{Re}\sum_{j=1}^{2} r_j^{(1)} A_j^{(1)}.$$

If $|z^{(2)}_j|$ are small, the functions $\Phi^{(2)}_j$ $(z^{(2)}_j)$ are put in the following form ($P^{(\alpha)}_z$ = 0):

$$\Phi_j^{(2)}\left(z_j^{(2)}\right) = A_j^{(2)} + B_j^{(2)} z_j^{(2)} + o\left(z_j^{(2)2}\right) \quad \left(z_j^{(2)} \in S_j^{(2)}, \; j = 1, 2\right).$$

When the condition $\sum_{k=1}^{N} k|C_k|^2 < 1$ is obeyed [4], the function

$$z = \omega(\zeta) = R\left(\zeta + \sum_{k=1}^{N} C_k \zeta^{-k}\right)$$

conformally maps the region outside unit circle $\gamma(|\zeta| \geq 1)$ into the region outside the line of division L described by (1).

The equations for the contours $L^{(\alpha)}_j$ of the regions $S^{(\alpha)}_j$ according to (1) and (5) take the form

$$t_j^{(\alpha)} = \sum_{k=1}^{N}\left(\lambda_k^{(\alpha j)}\sigma^k + \mu_k^{(\alpha j)}\sigma^{-k}\right) \quad (\sigma \in \gamma, \quad j, \alpha = 1, 2), \tag{7}$$

where

$$\lambda_1^{(\alpha j)} = \frac{1}{2}\left[R\left(1 - i\mu_j^{(\alpha)}\right) + \left(1 + i\mu_j^{(\alpha)}\right)\overline{RC_1}\right]; \quad \lambda_k^{(\alpha j)} = \frac{1}{2}\,\overline{RC_k}\left(1 + i\mu_j^{(\alpha)}\right);$$

$$\mu_1^{(\alpha j)} = \frac{1}{2}\left[\bar{R}\left(1 + i\mu_j^{(\alpha)}\right) + \left(1 - i\mu_j^{(\alpha)}\right)RC_1\right]; \quad \mu_k^{(\alpha j)} = \frac{1}{2}\,RC_k\left(1 - i\mu_j^{(\alpha)}\right)$$

$$(k = 2, \ldots, N).$$

From (7) we get

$$dt_j^{(\alpha)} = \left[\sum_{k=1}^{N} k\lambda_k^{(\alpha j)}\sigma^{k-1} - \sum_{k=1}^{N} k\mu_k^{(\alpha j)}\sigma^{-k-1}\right] d\sigma \quad (j = 1, 2). \tag{8}$$

In the transformed regions outside and within unit circle γ, the functions $\Phi^{(\alpha)}{}_j(z^{(\alpha)}{}_j)$ and the arbitrary holomorphic function $F(z)$ can be represented as power series in the variables $\zeta^{(\alpha)}{}_j$ $(j, \alpha = 1, 2)$ and ζ:

$$\Phi_j^{(1)}\left(z_j^{(1)}\right) = A_j^{(1)} + \sum_{k=1}^{\infty} A_k^{(1j)} \zeta_j^{(1)-k}, \quad \lim_{\left|z_j^{(1)}\right| \to \infty} \Phi_j^{(1)}\left(z_j^{(1)}\right) = A_j^{(1)},$$
$$\Phi_j^{(2)}\left(z_j^{(2)}\right) = A_j^{(2)} + B_j^{(2)} z_j^{(2)} + \sum_{k=1}^{\infty} A_k^{(2j)} \zeta_j^{(2)k}; \tag{9}$$

$$F(z) = F[\omega(\zeta)] = \sum_{n=0}^{\infty} E_n \zeta^{-n}$$
$$\left(z_j^{(\alpha)} \in S_j^{(\alpha)}, \quad z \in S^{(1)}; \quad \left|\zeta_j^{(1)}\right| \geqslant 1, \quad \left|\zeta_j^{(2)}\right| \leqslant 1, \quad j, \alpha = 1, 2\right). \tag{10}$$

and on unit circle γ the variables ζ and $\zeta^{(\alpha)}{}_j$ take the same value $\sigma = e^{i\theta}$.

On the basis of (8) and (9) we find for the boundaries $L^{(\alpha)}{}_j$ of the regions $S^{(\alpha)}{}_j$ $(z^{(\alpha)}{}_j \to t^{(\alpha)}{}_j,\ \zeta^{(\alpha)}{}_j \to \sigma)$ that the following representations apply:

$$\Phi_j^{(1)}\left(t_j^{(1)}\right) dt_j^{(1)} = A_j^{(1)} dt_j^{(1)} + \left[\sum_{k=0}^{N-2} a_k^{(1j)} \sigma^k + \sum_{k=1}^{\infty} b_k^{(1j)} \sigma^{-k}\right] d\sigma,$$
$$\Phi_j^{(2)}\left(t_j^{(2)}\right) dt_j^{(2)} = A_j^{(2)} dt_j^{(2)} + B_j^{(2)} t_j^{(2)} dt_j^{(2)} + \left[\sum_{k=0}^{\infty} a_k^{(2j)} \sigma^k + \sum_{k=1}^{N} b_k^{(2j)} \sigma^{-k}\right] d\sigma, \tag{11}$$

where

$$a_n^{(1j)} = \sum_{k=1}^{N-n-1} (k+n+1) \lambda_{k+n+1}^{(1j)} A_k^{(1j)},$$
$$b_n^{(1j)} = \sum_{k=1}^{N+n-1} (k-n+1) \lambda_{k-n+1}^{(1j)} A_k^{(1j)} - \sum_{k=1}^{n-2} (n-k-1) \mu_{n-k-1}^{(1j)} A_k^{(1j)},$$
$$a_n^{(2j)} = \sum_{k=1}^{n} (n-k+1) \lambda_{n-k+1}^{(2j)} A_k^{(2j)} - \sum_{k=1}^{N+n+1} (k-n-1) \mu_{k-n-1}^{(2j)} A_k^{(2j)},$$
$$b_n^{(2j)} = -\sum_{k=1}^{N-n+1} (k+n-1) \lambda_{k+n-1}^{(2j)} A_k^{(2j)}, \tag{12}$$

with $\lambda^{(\alpha j)}{}_n = 0$, $\mu^{(\alpha j)}{}_n = 0$ for $n > N$ $(j, \alpha = 1, 2)$. From (8) and (11) we get the boundary values of the functions

$$\Phi_j^{(\alpha)}\left(z_j^{(\alpha)}\right) \quad (j, \alpha = 1, 2) \text{ on } L_j^{(\alpha)} \text{ for } z_j^{(\alpha)} \to t_j^{(\alpha)}, \quad \zeta_j^{(\alpha)} \to \sigma \quad (\sigma \in \gamma)$$

$$\Phi_j^{(1)}\left(t_j^{(1)}\right) = A_j^{(1)} + \frac{\sum_{k=0}^{N-2} a_k^{(1j)} \sigma^k + \sum_{k=1}^{\infty} b_k^{(1j)} \sigma^{-k}}{\sum_{k=1}^{N} k \lambda_k^{(1j)} \sigma^{k-1} - \sum_{k=1}^{N} k \mu_k^{(1j)} \sigma^{-k-1}} \quad (j = 1, 2), \tag{13}$$

$$\Phi_j^{(2)}\left(t_j^{(2)}\right) = A_j^{(2)} + B_j^{(2)} t_j^{(2)} + \frac{\sum_{k=0}^{\infty} a_k^{(2j)} \sigma^k + \sum_{k=1}^{N} b_k^{(2j)} \sigma^{-k}}{\sum_{k=1}^{N} k \lambda_k^{(2j)} \sigma^{k-1} - \sum_{k=1}^{N} k \mu_k^{(2j)} \sigma^{-k-1}} \quad (j = 1, 2).$$

We substitute (4), (10), and (11) into (3) and integrate on the basis that the function of (10) is arbitrary to get a finite system of linear algebraic equations for the coefficients $a^{(\alpha j)}{}_n$, $b^{(\alpha j)}{}_n$, $A^{(2)}{}_j$, $B^{(2)}{}_j$ of the form

$$\sum_{j=1}^{2} \sum_{\alpha=1}^{2} (-1)^{\alpha+1} \left[\left(q_j^{(\alpha)} + i \frac{p_j^{(\alpha)}}{\mu_j^{(\alpha)}}\right)\left(a_{n-1}^{(\alpha j)} + n\lambda_n^{(\alpha j)} A_j^{(\alpha)} + \varepsilon_{n-1}^{(2j)} B_j^{(2)} \delta_{2\alpha}\right) - \left(\bar{q}_j^{(\alpha)} + i \frac{\bar{p}_j^{(\alpha)}}{\bar{\mu}_j^{(\alpha)}}\right)\left(\bar{b}_{n+1}^{(\alpha j)} - n\bar{\mu}_n^{(\alpha j)} \bar{A}_j^{(\alpha)} + \bar{\eta}_{n+1}^{(2j)} \bar{B}_j^{(2)} \delta_{2\alpha}\right)\right] = RC\delta_{n1} i,$$

$$\sum_{j=1}^{2} \sum_{\alpha=1}^{2} (-1)^{\alpha+1} \left[\left(\bar{q}_j^{(\alpha)} + i \frac{\bar{p}_j^{(\alpha)}}{\bar{\mu}_j^{(\alpha)}}\right)\left(\bar{a}_{n-1}^{(\alpha j)} + n\bar{\lambda}_n^{(\alpha j)} \bar{A}_j^{(\alpha)} + \bar{\varepsilon}_{n-1}^{(2j)} \bar{B}_j^{(2)} \delta_{2\alpha}\right) - \right.$$

$$\left(q_j^{(\alpha)} + i\frac{p_j^{(\alpha)}}{\mu_j^{(\alpha)}}\right)\left(b_{n+1}^{(\alpha j)} - n\mu_n^{(\alpha j)}A_j^{(\alpha)} + \eta_{n+1}^{(2j)}B_j^{(2)}\delta_{2\alpha}\right)\Bigg] = RnCC_n i, \tag{14}$$

$$\sum_{j=1}^{2}\sum_{\alpha=1}^{2}(-1)^{\alpha+1}\left[\left(1+i\mu_j^{(\alpha)}\right)\left(a_{n-1}^{(\alpha j)} + n\lambda_n^{(\alpha j)}A_j^{(\alpha)} + \varepsilon_{n-1}^{(2j)}B_j^{(2)}\delta_{2\alpha}\right) - \left(1+i\bar{\mu}_j^{(\alpha)}\right)\left(\bar{b}_{n+1}^{(\alpha j)} - n\bar{\mu}_n^{(\alpha j)}\bar{A}_j^{(\alpha)} + \bar{\eta}_{n+1}^{(2j)}\bar{B}_j^{(2)}\delta_{2\alpha}\right)\right] = 0,$$

$$\sum_{j=1}^{2}\sum_{\alpha=1}^{2}(-1)^{\alpha+1}\left[\left(1+i\bar{\mu}_j^{(\alpha)}\right)\left(\bar{a}_{n-1}^{(\alpha j)} + n\bar{\lambda}_n^{(\alpha j)}\bar{A}_j^{(\alpha)} + \bar{\varepsilon}_{n-1}^{(2j)}\bar{B}_j^{(2)}\delta_{2\alpha}\right) - \left(1+i\mu_j^{(\alpha)}\right)\left(b_{n+1}^{(\alpha j)} - n\mu_n^{(\alpha j)}A_j^{(\alpha)} + \eta_{n+1}^{(2j)}B_j^{(2)}\delta_{2\alpha}\right)\right] = 0 \quad (n = 1, \ldots, 2N),$$

where

$$\varepsilon_n^{(\alpha j)} = \sum_{k=1}^{n} k\lambda_k^{(\alpha j)}\lambda_{n-k+1}^{(\alpha j)} + \sum_{k=1}^{N} k\lambda_k^{(\alpha j)}\mu_{k-n-1}^{(\alpha j)} - \sum_{k=1}^{N-n-1} k\lambda_{k+n+1}^{(\alpha j)}\mu_k^{(\alpha j)};$$

$$\eta_n^{(\alpha j)} = \sum_{k=1}^{N-n+1} k\lambda_k^{(\alpha j)}\mu_{k+n-1}^{(\alpha j)} - \sum_{k=1}^{n-2} k\mu_k^{(\alpha j)}\mu_{n-k-1}^{(\alpha j)} - \sum_{k=1}^{N} k\lambda_{k-n+1}^{(\alpha j)}\mu_k^{(\alpha j)};$$

in which $\varepsilon^{(\alpha j)}{}_n = 0$ for $n > 2N - 1$, $\eta^{(\alpha j)}{}_n = 0$ for $n > 2N + 1$, δ_{nk} is the Kronecker symbol. The quantities $a^{(\alpha j)}{}_n$ and $b^{(\alpha j)}{}_n$ are expressed in terms of the coefficients $A^{(\alpha j)}{}_k$ ($j, \alpha = 1, 2$) by means of (12).

If the inclusion is free from external load, we have from (6) with (11) that

$$b_1^{(\alpha j)} = 0 \quad (j, \alpha = 1,2).$$

Also, outside unit circle γ ($\alpha = 1$) and within it ($\alpha = 2$) the complex potentials $\Phi^{(\alpha)}{}_j(z^{(\alpha)}{}_j)$ ($j, \alpha = 1, 2$) should be bounded. For this purpose we specify that the zeros in the numerator coincide with the zeros in the denominator in the fractionally rational part of the functions of (13), with the number of these outside γ being $N - 1$ and the number inside γ being $N + 1$ [4].

Then the coefficients $a^{(\alpha j)}{}_n$ and $b^{(\alpha j)}{}_n$ should satisfy the conditions

$$\sum_{k=0}^{N-2} a_k^{(1j)}\zeta_{jn}^{(1)k} + \sum_{k=1}^{2N+1} b_k^{(1j)}\zeta_{jn}^{(1)-k} = 0, \tag{15}$$

$$\sum_{k=0}^{2N-1} a_k^{(2j)}\zeta_{jl}^{(2)k} + \sum_{k=1}^{N} b_k^{(2j)}\zeta_{jl}^{(2)-k} = 0 \quad (n = \overline{1, N-1},\ \ l = \overline{1, N+1},\ \ j = 1, 2),$$

where $\zeta^{(1)}{}_{jn}$ and $\zeta^{(2)}{}_{jl}$ are the roots of the equations

$$\sum_{k=1}^{N} k\lambda_k^{(\alpha j)}\zeta_j^{(\alpha)k-1} - \sum_{k=1}^{N} k\mu_k^{(\alpha j)}\zeta_j^{(\alpha)-k-1} = 0 \tag{16}$$

correspondingly greater than one and less than one in modulus:

$$\left|\zeta_{jn}^{(1)}\right| > 1, \quad \left|\zeta_{jl}^{(2)}\right| < 1 \qquad (n = \overline{1, N-1},\ l = \overline{1, N+1},\ j = 1, 2).$$

We combine conditions (15) with (14) to get a closed system of linear algebraic equations of order 12N for the coefficients

$$a_{k-1}^{(1j)},\ b_{k+1}^{(2j)},\ a_{n-1}^{(2j)},\ b_{n+1}^{(1j)},\ A_j^{(2)},\ B_j^{(2)} \qquad (k = \overline{1,\ N-1},\ \ n = \overline{1,\ 2N},\ \ j = 1, 2).$$

For N = 1 we get a solution for a plate bearing an elliptic inclusion. Then, as follows from (14)-(16) we have $a_k^{(\alpha j)} = 0,\ b_k^{(2j)} = 0,\ B_j^{(2)} = 0,\ b_2^{(1j)} \neq 0,\ b_m^{(1j)} = 0\ (m > 2)$ and the functions of (13) take the form

$$\Phi_j^{(1)}\left(z_j^{(1)}\right) = A_j^{(1)} + \frac{b_2^{(1j)}}{\lambda_1^{(1j)}\zeta_j^{(1)2} - \mu_1^{(1j)}}\left(z_j^{(1)} \to t_j^{(1)},\ \zeta_j^{(1)} \to \sigma\right), \quad \Phi_j^{(2)}\left(z_j^{(2)}\right) = A_j^{(2)},$$

where

$$\zeta_j^{(1)} = \frac{z_j^{(1)} + \sqrt{z_j^{(1)2} - 4\mu_1^{(1j)}\lambda_1^{(1j)}}}{2\lambda_1^{(1j)}}; \quad \lambda_1^{(1j)} = \frac{a - i\mu_j^{(1)}b}{2};$$

$\mu_1^{(1j)} = \dfrac{a + i\mu_j^{(1)}b}{2}$; $C_1 = \dfrac{(a-b)}{(a+b)}$; and a and b are the semiaxes of the ellipse.

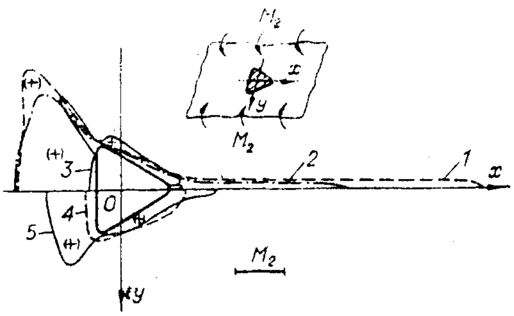

Fig. 3

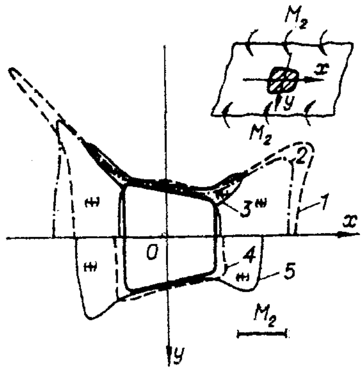

Fig. 4

As an example we consider the bending of an orthotropic plate bearing an elastic orthotropic inclusion, where the line of junction is described by

$$z = \omega(\sigma) = R(\sigma + C_1\sigma^{-1} + C_2\sigma^{-2} + C_3\sigma^{-3}).$$

By varying the coefficients C_k, one can obtain an equation for the line of junction, for example in the form of a triangle ($C_1 = 0$, $C_2 = 0.25$, $C_3 = 0$), trapezium ($C_1 = -0.021$, $C_2 = 0.102$, $C_3 = -0.150$), or rectangle ($C_1 = 0.632$, $C_2 = 0$, $C_3 = -0.099$ for the ratio of the size of the rectangle $\lambda = a_1/b_1 = 5$ [5]) with rounded corners, etc. [6].

At infinity the plate is bent by the moments $M^\infty_x = M_1$, $M^\infty_y = M_2$, $H^\infty_{xy} = 0$. The inclusion is free from external load. The principal elasticity directions in the plate and inclusion are parallel to the coordinate axes x and y ($\varphi = 0$). In that case,

$$M_x^{(\alpha)} = 0,\quad M_y^{(\alpha)} = 0,\quad P_z^{(\alpha)} = 0,\quad \mu_2^{(\alpha)} = -\bar{\mu}_1^{(\alpha)},\quad p_2^{(\alpha)} = \bar{p}_1^{(\alpha)},\quad q_2^{(\alpha)} = \bar{q}_1^{(\alpha)},\quad r_2^{(\alpha)} = -\bar{r}_1^{(\alpha)},\quad a_n^{(\alpha 2)} = \bar{a}_n^{(\alpha 1)},\quad b_n^{(\alpha 2)} = \bar{b}_n^{(\alpha 1)},$$

$$A_2^{(2)} = \bar{A}_1^{(2)},\quad B_2^{(2)} = \bar{B}_1^{(2)},\quad R = \bar{R},\quad C_k = \bar{C}_k,\quad \lambda_k^{(\alpha 2)} = \bar{\lambda}_k^{(\alpha 1)},\quad \mu_k^{(\alpha 2)} = \bar{\mu}_k^{(\alpha 1)},\quad \varepsilon_n^{(\alpha 2)} = \bar{\varepsilon}_n^{(\alpha 1)},$$

$$\eta_n^{(\alpha 2)} = \bar{\eta}_n^{(\alpha 1)},\quad A_1^{(1)} = \frac{\bar{q}_1^{(1)} M_x^\infty - \bar{p}_1^{(1)} M_y^\infty}{2\left(\bar{p}_1^{(1)} q_1^{(1)} - p_1^{(1)} \bar{q}_1^{(1)}\right)},\quad A_2^{(1)} = \bar{A}_1^{(1)},\quad b_1^{(\alpha j)} = 0 \quad (j, \alpha = 1, 2)$$

(all the coefficients are complex).

The numerical calculation was performed for an orthotropic plywood plate and an inclusion with the following characteristics [1]:

$$\text{(A)}\ E_x = E_{max},\ E_1 = 0{,}165\cdot 10^5\ \text{MPa},\ E_2 = 0{,}137\cdot 10^4\ \text{MPa},$$
$$G = 0{,}686\cdot 10^3\ \text{MPa},\ \nu_1 = 0{,}31,\ \nu_2 = 0{,}026,\ \mu_1 = \alpha + i\beta,$$
$$\mu_2 = -\bar{\mu}_1,\ \alpha = 1{,}04,\ \beta = 1{,}55;$$
$$\text{(B)}\ E_x = E_{min},\ E_1 = 0{,}137\cdot 10^4\ \text{MPa},\ E_2 = 0{,}165\cdot 10^5\ \text{MPa},$$
$$G = 0{,}686\cdot 10^3\ \text{MPa},\ \nu_1 = 0{,}026,\ \nu_2 = 0{,}31,\ \mu_1 = \alpha + i\beta,$$
$$\mu_2 = -\bar{\mu}_1,\ \alpha = 0{,}299,\ \beta = 0{,}444.$$

Figure 2 shows the distribution of the moments $M^{(\alpha)}_\theta$ ($\alpha = 1, 2$) in the plate (A) containing a rectangular ($a_1/b_1 = 5$) inclusion (B) along the line of junction. The line denoted by 1 characterizes the bending moments $M^{(1)}_\theta$ in a plate bearing a hole, while line 2 is for a plate with an elastic core, 3 is for a plate with a rigid core, 4 represents the bending moments $M^{(2)}_\theta$ in the core, and 5 represents the bending moments when the plate and core are of a single material. The same graphs are also given in Figs. 3 and 4 for a plate (B) correspondingly with the above triangular and trapezoidal inclusions (A).

When the principal elasticity directions of the plate and inclusion lie at an arbitrary angle φ, we have to rotate the x'y'z' coordinate system around the axis $z \equiv z'$ through an angle φ (Fig. 1, with the x' and y' axes coincident with the principal elasticity directions in the inclusion) and convert the rigidities $D^{(2)}_{ij}$ of the inclusion on going to the new axes in accordance with formulas that take the following form [1] for an orthotropic material:

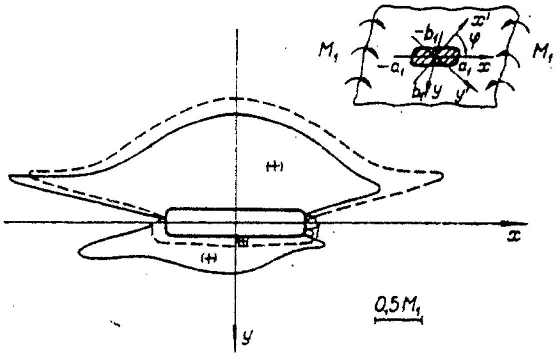

Fig. 5

$$D_{11}^{(2)} = D_1^{(2)} \cos^4 \varphi + 2D_3^{(2)} \sin^2 \varphi \cos^2 \varphi + D_2^{(2)} \sin^4 \varphi,$$
$$D_{22}^{(2)} = D_1^{(2)} \sin^4 \varphi + 2D_3^{(2)} \sin^2 \varphi \cos^2 \varphi + D_2^{(2)} \cos^4 \varphi,$$
$$D_{66}^{(2)} = D_k^{(2)} + \left(D_1^{(2)} + D_2^{(2)} - 2D_3^{(2)}\right) \sin^2 \varphi \cos^2 \varphi,$$
$$D_{12}^{(2)} = D_2^{(2)} \nu_1^{(2)} + \left(D_1^{(2)} + D_2^{(2)} - 2D_3^{(2)}\right) \sin^2 \varphi \cos^2 \varphi,$$
$$D_{16}^{(2)} = \frac{1}{2}\left(D_2^{(2)} \sin^2 \varphi - D_1^{(2)} \cos^2 \varphi + D_3^{(2)} \cos 2\varphi\right) \sin 2\varphi,$$
$$D_{26}^{(2)} = \frac{1}{2}\left(D_2^{(2)} \cos^2 \varphi - D_1^{(2)} \sin^2 \varphi - D_3^{(2)} \cos 2\varphi\right) \sin 2\varphi.$$

The following formulas [1] are used in converting the complex bending parameters $\mu^{(2)}_1$ and $\mu^{(2)}_2$ of the inclusion on going to the new axes:

$$\mu_1^{(2)} = \frac{\mu_1^{(2)\prime} \cos \varphi - \sin \varphi}{\cos \varphi + \mu_1^{(2)\prime} \sin \varphi}, \quad \mu_2^{(2)} = \frac{\mu_2^{(2)\prime} \cos \varphi - \sin \varphi}{\cos \varphi + \mu_2^{(2)\prime} \sin \varphi}.$$

Here $D_1^{(2)}$, $D_2^{(2)}$, $D_k^{(2)}$, $D_3^{(2)} = D_1^{(2)} \nu_2^{(2)} + 2D_k^{(2)}$; $\mu_1^{(2)\prime}$, $\mu_2^{(2)\prime}$ are the principal rigidities and complex bending parameters for the orthotropic inclusion in the x'y'z' coordinate system (Fig. 1).

Figure 5 shows graphs for the distribution of the moments $M^{(\alpha)}_\theta$ (α = 1, 2) on the line of junction between the plate (A) and a rectangular inclusion (a_1/b_1 = 5) (B) when the principal elasticity directions of the plate and inclusion lie at a mutual angle $\varphi = \pi/3$ (solid line). The graphs in the upper part of Fig. 5 show the bending moments in the plate, while those in the lower part show those in the core. The broken lines characterizes the case $\varphi = 0$.

LITERATURE CITED

1. S. G. Lekhnitskii, Anisotropic Plates [in Russian], GITTL, Moscow (1957).
2. G. N. Savin, Stress Concentration around Holes [in Russian], GITTL, Moscow—Leningrad (1951).
3. T. L. Martynovich, "An exact solution for the bending of an anisotropic plate containing a hole," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1977).
4. I. I. Privalov, Introduction to the Theory of Functions of the Complex Variable [in Russian], Gostekhizdat, Moscow (1954).
5. A. S. Kosmodamianskii, Planar Problems in the Theory of Elasticity for Plates with Holes, Notches, and Ridges [in Russian], Vishcha Shkola, Kiev (1975).
6. G. N. Savin, Mechanics of Deformable Media: Selected Works [in Russian], Naukova Dumka, Kiev (1979).